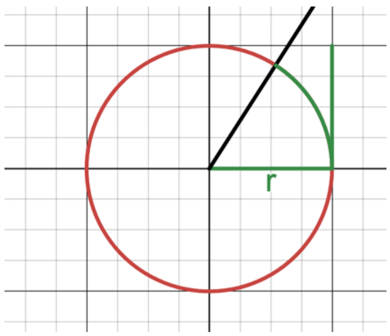
5.1 The Unit Circle

Radian Measure Revisited

We said 1 revolution = _______________. (arbitrarily chosen at the time)

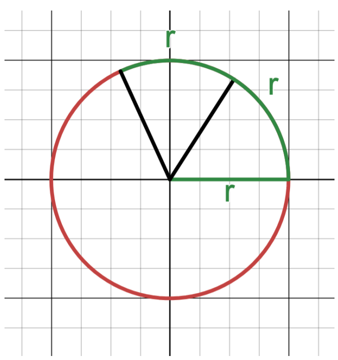
But why 2π? What is a radian?

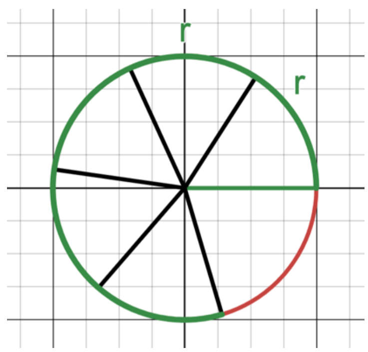


Definition:
One radian is defined as the angle subtended from the center of a circle which intercepts an arc equal in length to the radius (see animation https://www.youtube.com/watch?v=MgbKapLpgUA)

So the measure of θ is ________________?

How many radians does it take to go all the way around?____________________

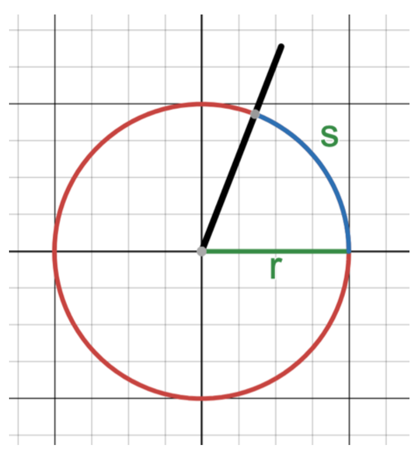




Arc Length

Proportions give us a relationship between the length, s, of an arc and the measure, θ, of an angle measured

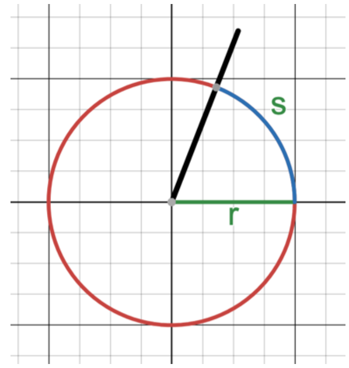
in ________________



<u>Unit Circle.</u>

When discussing angles in radians, we often consider the "Unit Circle". $x^2 + y^2 = 1$

What is special about this circle?

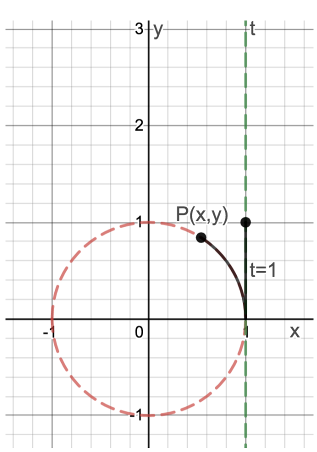


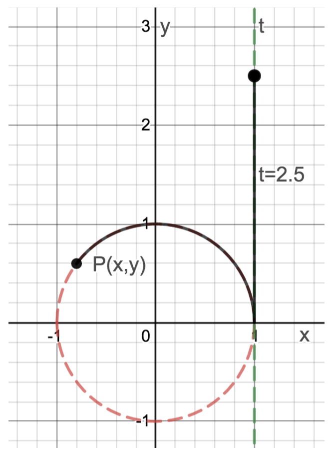
Are the following points on the unit circle? $A\left(\frac{3}{5},\frac{4}{5}\right)$ $B\left(\frac{1}{2},\frac{-2}{3}\right)$

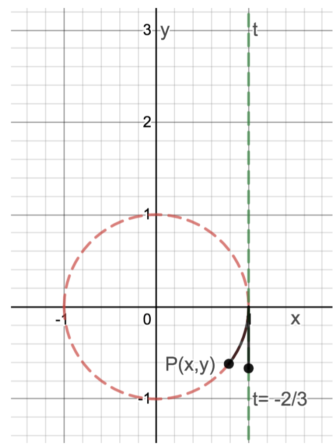
Find a point on the unit circle given the following conditions:

Ex. The point $P\left(\frac{1}{3},y\right)$ is on the unit circle in Quadrant IV. Find y.

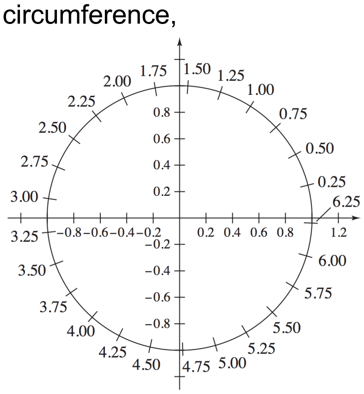
<u>Exploring Arc Length: "Unit Circle Wrap Process"</u>
Consider the real number line corresponding values of t aligned next to the unit circle as shown where positive values of t are shown upward, negative are downward. If this number line were wrapped around the unit circle, then every number t would correspond to a point P(x,y) on the unit circle found by using $|t|$ as the arc length. (Notice: Angles are not being discussed here)







We can approximate the coordinates for a given arc length using a unit circle with units marked around the circumference,



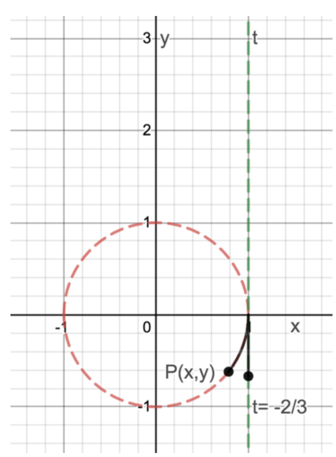
How would you (roughly) find the *location* (don't worry about coordinates yet) of the point corresponding to without using the above graph?:

a). $t = 3$?

b) $t = \pi / 3$?

c) $t = -11\pi / 6$

d) $t = 9\pi / 8$



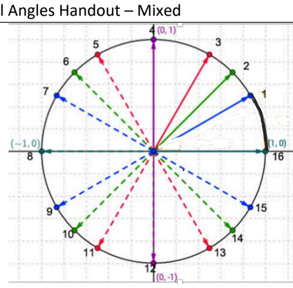
So given any real number, we can find the location of its terminal point on the unit circle by

____

In this context, we use the terminology “reference number” instead of “reference angle”

We have been doing this, but thinking in terms of ____________________

Angle Worksheet 3 -Special Angles Handout – Mixed



Given that all the “blue angles” have a reference angle of 30 degrees or $\pi/6$ radians, write the angle measures, both in radians AND degrees, for each of the blue angles.

1) 30° $\pi/6$

7) 150° $5\pi/6$

9) 210° $7\pi/6$

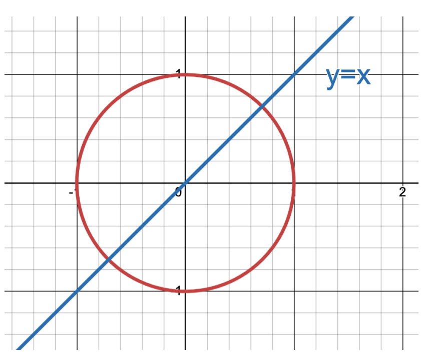
How do we find actual exact values of those points?

Find the (exact) terminal point for the real number:

$t = \pi$ $\qquad$ $t = \pi / 2$ $\qquad$ $t = -6\pi$

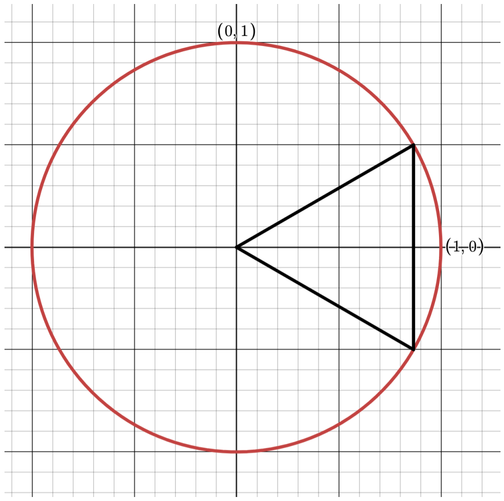
For what other real numbers (angles) can we find exact values:

$t = \pi / 4$



$t = \pi / 6$

$t = \pi / 3$



Summarizing

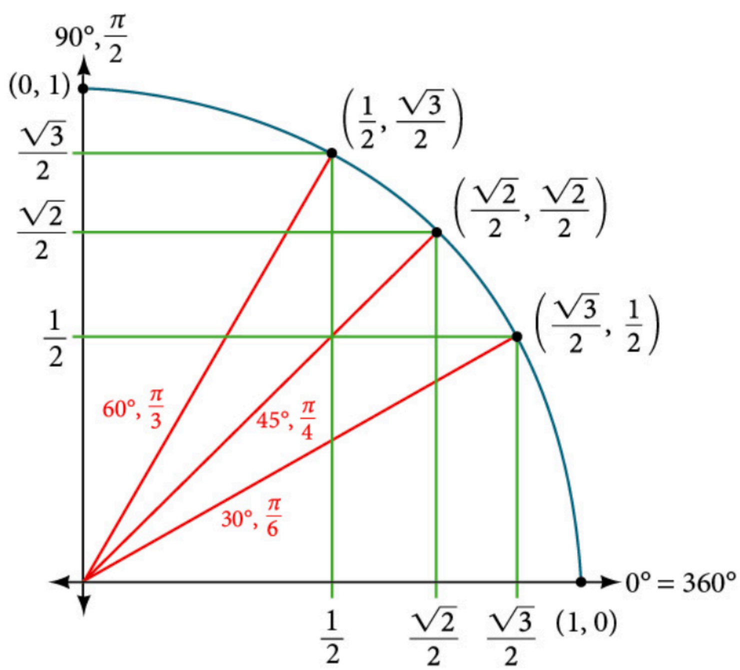
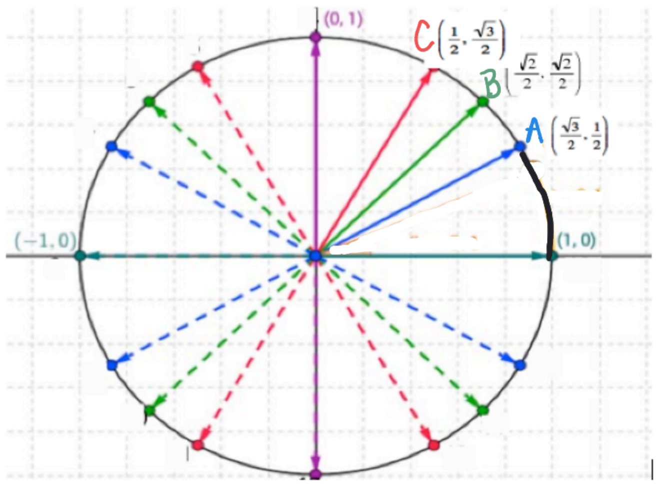


Figure 14

Symmetry and Important Points on the Unit Circle.

We are often interested in looking where the terminal side of some of the "key numbers t" (or angles θ) mentioned earlier intersect the "unit circle" $x^2 + y^2 = 1$. We have now found the points in the first quadrant shown.



Again, answer the following:

Coordinates of point which is symmetric to A relative to the y axis____

Coordinates of point which is symmetric to A relative to the x axis___

Coordinates of point which is symmetric to A relative to the origin.___

List the points in ascending order according to their y value. ___________

List the points in ascending order according to their x value. ___________

Example:

Find the point on the unit circle corresponding to $t = \frac{5\pi}{3}$ ____________________

Find the y value of the point on the unit circle corresponding to $t = \frac{5\pi}{6}$

Find the x value of the point on the unit circle corresponding to $t = \frac{-\pi}{4}$ ____________________

Find the coordinates of the point on the unit circle corresponding to $t = \frac{4\pi}{3}$ ____________________